Grade 5 to 8 Mathematics

Classroom-Based Assessment

Manitoba Education, Training and Youth

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INTRODUCTION

This document will provide teachers with practical strategies for classroom-based assessment.

To implement the Manitoba mathematics curriculum and to ensure that students successfully master the prescribed learning outcomes, teachers must have a plan for instruction and assessment. Planning needs to take place at both the school and the classroom levels. The school should be seen as a community of learners—both staff and students—learning together. Communication among teachers of various grades is also important to facilitate discussion about grade-level outcomes, student progress related to those outcomes, and strategies for the continued success of all students. Without this communication, teachers are repeatedly starting over with each new class of students. School-based planning helps teachers to see the big picture: the role of mathematics in the whole school and in the community. Using classroom-based assessments and any other testing/assessment tools that are available, teachers are able to see general strengths and weaknesses of both individual students and the program.

Curriculum changes, increased or changing student needs, research related to how students learn, and a rapidly changing society are just some of the challenges teachers face today. Teachers cannot do everything themselves. There has never been a greater need for cooperation and collaboration among teachers at all grade levels than there is now in the 21st century.

Cooperative planning allows teachers to share the work. One person can work on the teaching strategies and assessment tasks for a given strand or section of a strand; other team member(s) can work on other strands or sections. In this way, a person is able to focus on a narrow portion of the curriculum (perhaps an area he or she considers a strength) and develop a comprehensive plan. If this type of planning is not possible, then sharing ideas and assessment tasks can be very beneficial. It is important that students have the opportunity to experience assessment tasks from a variety of sources. Students become comfortable with the presentation format used by their classroom teachers. They need to be exposed to presentation formats used by other teachers. All students do not respond in the same way to the same lesson; exposing them to various presentation formats takes this into consideration.

Mathematics Classroom Assessment

Assessment is a "systematic process of gathering information about what a student knows, is able to do, and is learning to do" (Manitoba Education and Training, *Reporting on Student Progress and Achievement* 5). Assessment is an integral part of instruction that enhances, empowers, and celebrates student learning.

Meaningful Assessment

The purpose of meaningful assessment is to inform instruction by providing information about student learning. This information can then be used to provide direction for planning further instruction. Assessment should occur in authentic contexts that allow students to demonstrate learning by performing meaningful tasks.

Meaningful content and contexts for assessment help students by engaging their attention and encouraging them to share their work and talk about their progress. Students need to take an active part in assessment. When students understand assessment criteria and procedures, and take ownership for assessing the quality, quantity, and processes of their own work, they develop self-assessment skills. The ultimate goal of assessment is to help develop independent, life-long learners who regularly monitor and assess their own progress.

The Teacher's Role in Assessment

In the classroom, teachers are the primary assessors of students. Teachers design assessment tools with two broad purposes: to collect information that will inform classroom instruction, and to monitor students' progress toward achieving year-end mathematics learning outcomes. Teachers also assist students in developing self-monitoring and self-assessment skills and strategies. To do this effectively, teachers must ensure that students are involved in setting learning goals, developing action plans, and using assessment processes to monitor their achievement of goals. Teachers also create opportunities to celebrate their progress and successes.

Teachers learn about student learning and progress by regularly and systematically observing students in action, and by interacting with students during instruction. Because students' knowledge, and many of their skills, strategies, and attitudes are internal processes, teachers gather data and make judgments based on observing and assessing students' interactions, performances, and products or work samples. Teachers demonstrate that assessment is an essential part of learning. They model effective assessment strategies and include students in the development of assessment procedures, such as creating rubrics or checklists.

Assessment Purposes and Audiences

The quality of assessment largely determines the quality of evaluation. Evaluation is "the process of making judgments and decisions based on the interpretation of evidence gathered through assessment" (Manitoba Education and Training, *Reporting on Student Progress and Achievement* 39). Valid judgments can be made only if accurate and complete assessment data are collected in a variety of contexts over time. Managing assessment that serves a multitude of purposes and audiences is a challenging task. Teachers must continually balance the assessment of their students' progress in the development of knowledge, skills, strategies, and attitudes, with the purposes and audiences for the information collected.

ASSESSMENT TO ASSIST LEARNING AND INFORM INSTRUCTION

Principles of Assessment

1. An Integral Part of Instruction and Learning	2. Continual and Ongoing	3. Authentic and Meaningful Mathematics Learning and Contexts
Assessment... • is meaningful to students • leads to goal setting • fosters transfer/integration with other curricular areas and application to daily life • reflects instructional strategies used • uses a wide variety of strategies and tools • reflects a definite purpose	Assessment... • occurs through all instructional activities • occurs systematically over a period of time • demonstrates progress toward achievement of learning outcomes	Assessment... • focuses on connecting prior knowledge and new knowledge (integration of information) • focuses on authentic problem-solving contexts and tasks • focuses on application of strategies for constructing meaning in new contexts
4. Collaborative and Reflective Process	**5. Multi-dimensional—Incorporating a Variety of Tasks**	**6. Developmentally and Culturally Appropriate**
Assessment... • encourages meaningful student involvement and reflection • involves parents as partners • reaches out to the community • focuses on collaborative review of products and processes to draw conclusions • involves a team approach	Assessment... • uses a variety of authentic strategies, tasks, and tools • serves a variety of purposes and audiences • reflects instructional tasks	Assessment... • is suited to students' developmental levels • is sensitive to diverse social, cultural, and linguistic backgrounds • is unbiased
7. Focused on Students' Strengths	**8. Based on How Students Learn**	**9. Offer Clear Performance Targets**
Assessment... • identifies what students can do and are learning to do • identifies competencies in the development of knowledge, skills and strategies, and attitudes • considers preferred learning styles • focuses on celebrations of progress and success • provides for differentiation • provides information to compare a student's performance with his or her other performances	Assessment... • uses sound educational practice based on current learning theory and brain research • fosters development of metacognition • considers multiple intelligences and learning styles • uses collaborative and cooperative strategies • considers research on the role of memory in learning • reflects current models of mathematics learning	Assessment... • encourages student involvement (setting criteria, measuring progress, working toward outcomes and standards) • encourages application beyond the classroom • provides a basis for goal setting • provides students with a sense of achievement • provides information that compares a student's performance to predetermined criteria or standards

Note: These principles of assessment apply to all subject areas and grade levels.

* Adapted from Manitoba Education and Training, *Kindergarten to Grade 4 English Language Arts: A Foundation for Implementation* (Winnipeg, MB: Manitoba Education and Training, 1998) 24.

Planning for Instruction and Classroom Assessment Using Learning Outcomes

Learning outcomes and standards assist classroom teachers and other educators to

- plan learning activities that support student achievement
- establish goals for learning, instruction, and assessment
- monitor student progress in achieving learning outcomes and standards
- communicate with students, parents, and guardians about student progress
- develop a mathematics plan for a school

Planning a Balanced Mathematics Program

Developing a balanced, integrated mathematics program is a dynamic process. The program is shaped by the teaching style and resources of each teacher, by the interests and abilities of the students, and by the needs of the community.

Planning a balanced mathematics program needs to take into account that

- specific learning outcomes stated are end-of-year learning outcomes—students may achieve the learning outcomes at any time during the year
- students need practice in many meaningful contexts to consolidate new knowledge, skills, and strategies because learning is recursive and cumulative
- planning is continual, informed by ongoing classroom assessment
- a variety of instructional approaches, classroom management techniques, assessment practices, tools and strategies, and problem-solving activities are essential

Some areas of balance to consider in planning are

- learning outcomes for each grade
- four strands: Patterns and Relations, Statistics and Probability, Shape and Space, and Number
- standards of student performance for Grade 6
- student grouping patterns: individual, pairs, small groups, large groups, whole class, heterogeneous, homogeneous, student-directed, teacher-directed
- various learning styles and multiple intelligences
- various rates of student learning, addressed by providing pre-teaching, review, additional practice for some students, and challenging extension activities for others

Teachers strive for balance in their classrooms. The following diagram, "Planning to Ensure Balance," illustrates options to consider in planning instructional and assessment activities.

Planning to Ensure Balance*

explicit instruction	and	indirect instruction
interactive learning	and	independent learning
vicarious experiences	and	concrete experiences
teacher choices	and	student choices
teacher assessment	and	self/peer assessment
teacher talk	and	student talk

Planning for balance while ensuring sufficient instruction and practice in all outcomes defined for a particular grade is a challenging task. Looking at how the outcomes are related across grade levels is helpful (See Manitoba Education and Training, *Grades 5 to 8 Mathematics: Manitoba Curriculum Framework of Outcomes and Grade 6 Standards*). This information can be used to create balanced instructional activities and to help define areas in which initial teaching and practice are necessary and those in which practice and maintenance through problem-solving activities are the main emphases. It is important to note that students who have not reached maintenance levels of specific skills will need to continue with scaffolded instruction at their level. Scaffolding refers to "instructional approaches or strategies that provide adjustable and temporary assistance or support to the student in his or her achievement of the learning task" (Manitoba Education and Training, *Success For All Learners* 12.6).

Learning sequences, integrated units, and themes provide opportunities for explicit instruction of many learning outcomes. Instructional activities such as mini-lessons are necessary to introduce, develop, or reinforce particular skills. In every planning decision, reflective teachers need to ask:

- What is an appropriate balance for my students?
- Am I achieving the balance in my classroom, both in the short term and the long term?
- Is my instruction helping students achieve the appropriate learning outcomes of my mathematics program?

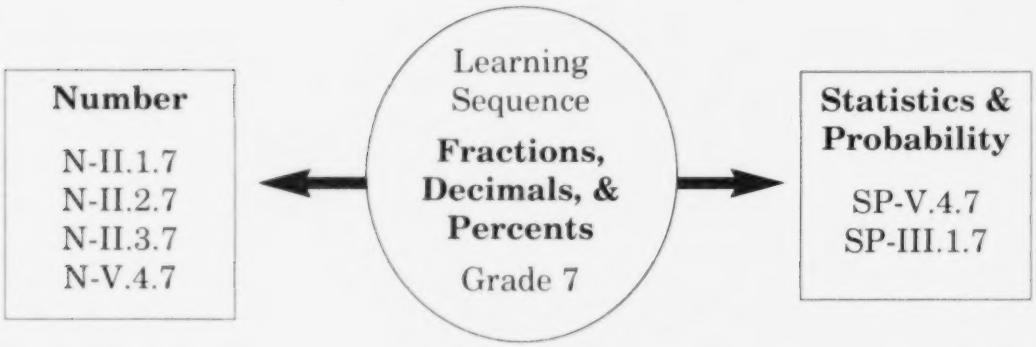
* Adapted from Manitoba Education and Training, *Kindergarten to Grade 4 English Language Arts: A Foundation for Implementation* (Winnipeg, MB: Manitoba Education and Training, 1998) 29.

It is important to know whether all of the learning outcomes for a particular grade level are being addressed. In order to do this, teachers might find it beneficial to make a list or an outcome map listing the learning outcomes for a specific grade level. Many learning outcomes need to be addressed repeatedly and in different ways throughout the year. When used in conjunction with daily planning, learning outcome maps help teachers assess whether all learning outcomes have been addressed in sufficient depth for all students to be successful. They also assist teachers in tracking and monitoring the balance of each strand in the mathematics curriculum. Learning outcome maps may also be used in conjunction with learning outcomes from other disciplines to assist in interdisciplinary planning. As planning takes place, outcomes that are addressed may be highlighted or underlined. In this way, teachers can see whether certain outcomes have been overlooked or have been revisited many times.

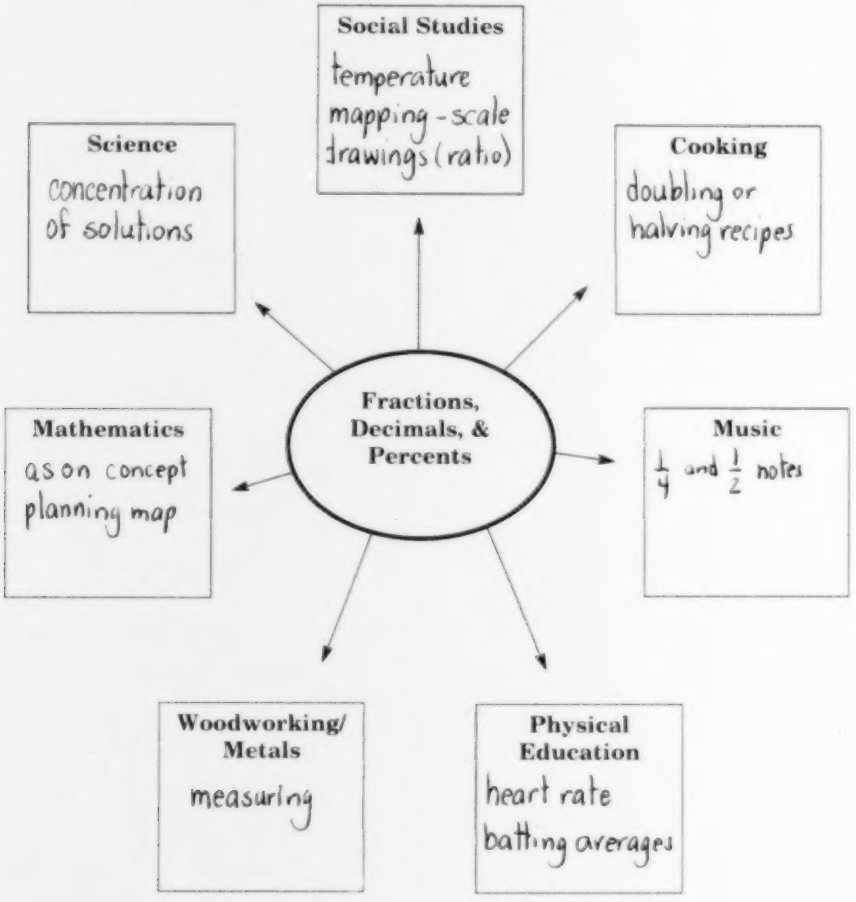
Examples: Checklist

Learning Outcome	Formative Assessment	Summative Assessment
designs and constructs rectangles of a given area, using natural numbers (SS-II. 2.6)	Nov 13 investigation Nov. 28 notebook entry Dec. 10	Dec 14 paper + pencil
demonstrates concretely, pictorially, and symbolically that many rectangles are possible for a given area or a given perimeter (SS-II.3.6)	Nov 16 Nov. 21 Nov 29 performance task Dec 4	Dec 14 interview

Concept Planning Map – Strand Integration



Concept Planning Map – Subject Integration



When planning to achieve learning outcomes, it is important to read each specific learning outcome carefully and clarify the types of learning that are expected of the student. This will provide guidance for developing appropriate instructional activities to help students meet the expectations of the mathematics curriculum.

Note: The Curriculum Information Technology Integration (CITI) project will allow teachers to integrate their planning in mathematics, science, and language arts.

Before teachers choose assessment tasks, tools, and strategies, it is critical to define the reasons for assessing, the audience who will use the information, and the ways results will be used. Appropriate assessment activities must focus on tasks that allow students to demonstrate their competence in applying learning in authentic ways.

Questions to Guide Results-Based Planning

1. *What do we want students to know and be able to do?*
   This question can be answered by reading the learning outcome to determine knowledge, skills, and strategies that it includes.
2. *What do students already know?*
   This question can be answered by having students work through activities that demonstrate their prior knowledge related to particular outcomes. Using strategies such as KWL Charts, word cycles, and concept maps would also help to provide information on prior knowledge.
3. *What instructional methods, materials, and strategies will help students develop these competencies?*
   These will be drawn from teachers' experiences, professional resources, or instructional suggestions provided in *Grades 5 to 8 Mathematics: A Foundation for Implementation* (Manitoba Education and Training).
4. *What is the purpose for assessment? How will the assessment be used?*
   Various assessment audiences and purposes are suggested in the chart "Principles of Assessment" on page 3.
5. *What assessment tasks will allow students to demonstrate their understanding in authentic ways?*
   Assessment tasks will be drawn from teachers' experiences, professional resources, or suggestions provided in *Grades 5 to 8 Mathematics: A Foundation for Implementation* (Manitoba Education and Training).

Planning with the End in Mind

To plan effectively, teachers need to have a clear sense of the purpose of instruction. The primary purpose of mathematics education is for the students to attain the mathematics learning outcomes identified and mandated for each grade level. The learning outcomes are not a checklist of what the teacher will teach. They are a description of the knowledge, skills and strategies, and attitudes that students are expected to demonstrate.

Placing the focus on student learning in this way means that all instructional planning must begin with students' present levels of performance. Teachers cannot decide on the first priorities for instruction until they know what knowledge, skills and strategies, and attitudes students bring into the class. Teachers are better able to make appropriate choices of topics, learning resources, groupings, and instructional strategies when they know the students as individuals. For this reason, teachers need to gather as much information about students as possible.

Using Initial Assessment for Planning

Before beginning work on a group of learning outcomes, teachers should gather information about ways students learn—both as a class and as individuals. Reflecting on these data is an essential part of the process of making decisions for future instruction, assessment, and student learning. Teachers need to ask:

- What have I learned about the class as a whole?
- What implications does this have for instruction? For assessment?
- For which students are these generalizations not accurate?
- How do I adapt instruction to meet the learning requirements of those particular students?

Purposes of Initial Assessment

Initial assessment helps teachers decide

- how much time to allocate to various learning outcomes
- which learning outcomes to focus on first
- what materials and learning resources are most appropriate
- which teaching and learning strategies to use

Phases of Learning

When preparing instructional plans and goals, teachers should consider three learning phases:

- activating (preparing for learning)
- acquiring (integrating and processing learning)
- applying (consolidating learning)

These phases are not entirely linear, but are a useful way of thinking and planning. A variety of activating, acquiring, and applying strategies are discussed in *Success for All Learners* (Manitoba Education and Training).

Preparing for Learning

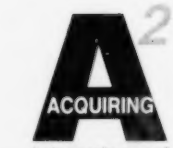
Activating (Preparing for Learning)

One of the strongest indications of how well students will comprehend new information is their prior knowledge of the subject. Some educators observe that more student learning occurs during the activating phase than at any other time. In planning instruction and assessment, teachers develop activities and select strategies for activating students' prior knowledge. These activities provide information about the extent of students' prior knowledge of the topic to be studied and their knowledge of and proficiency in applying skills and strategies needed for learning in this topic area.

Prior knowledge activities

- help students relate new information, skills, and strategies to what they already know and can do
- allow teachers to correct misconceptions that might otherwise persist and make learning difficult for students

- allow teachers to augment and strengthen students' knowledge base in cases where students do not possess adequate prior knowledge or experience to engage with new information and ideas
- help students recognize gaps in their knowledge
- stimulate curiosity, and initiate the inquiry process that will direct learning



Integrating and Processing Learning

Acquiring (Integrating and Processing Learning)

In the second phase of learning, students engage with new information and integrate it with what they already know, adding to and revising their previous knowledge. Part of the teacher's role in this phase is to present this new information, or to help students access it from other sources.

Since learning is an internal process, however, facilitating learning requires more of teachers than simply presenting information. In the acquiring phase, teachers instruct students in strategies that help them make meaning of information, integrate it with what they already know, and express their new understanding. In addition, teachers monitor these processes to ensure that learning is taking place, using a variety of instruments, tools, and strategies such as observations, interviews, and examination of student work.

Consolidating Learning

Applying (Consolidating Learning)

New learning that is not reinforced is soon forgotten. Teachers need to move students beyond guided practice and into independent practice. The products and performances by which students demonstrate new learning are not simply required for assessment; they play an essential instructional role in providing students with opportunities to demonstrate and consolidate their new knowledge, skills and strategies, and attitudes.

Students also need opportunities to reflect on what they have learned and to consider how new learning applies to new situations. By restructuring information or integrating what they have learned in one strand with other strands or subject areas, students strengthen and extend learning.

Purposes of Ongoing Assessment*

Ongoing assessment helps teachers decide

- whether students have mastered certain learning outcomes
- whether they are making progress in attaining other learning outcomes
- which learning outcomes need to be the focus of further instruction and assessment
- whether instructional resources, activities, and strategies need to be modified
- what tools would be most appropriate for assessment
- whether individual students need alternative learning experiences or further support

* Adapted from Manitoba Education and Training, *Senior 2 English Language Arts: A Foundation for Implementation* (Winnipeg, MB: Manitoba Education and Training, 1998) Planning–19.

Formative Assessment

Formative assessment is data collected about the whole group and/or individual students during classroom instruction.

Formative assessment is designed to guide instruction and improve student learning. This is done by

- identifying specific learning needs
- providing feedback describing students' performance

The instruments used in formative assessment provide information or data that teachers, parents, and students may use to identify factors that facilitate or hinder student learning.

Possible assessment strategies/tools that can be used for formative assessment include

- observations recorded on checklists or in teacher notes
- performance tasks with scoring rubrics
- diagnostic interviews
- group/peer assessments
- self-assessment
- paper-and-pencil tasks
- student journal/learning log entries

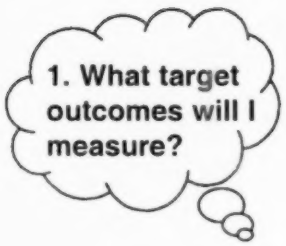
Note: "The thrust of formative assessment is toward improving learning and instruction. Therefore, the information should not be used for assigning marks, as the assessment often occurs before students have had full opportunities to learn content or develop skills" (Manitoba Education and Training, *Reporting on Student Progress and Achievement* 9–10).

Summative Assessment

Summative assessment occurs after students have had an opportunity to practice their learning independently. It often occurs during reporting times and is used to describe individual progress related to meeting the learning outcomes.

A GRADE 7 TEACHER'S PLANNING AND ASSESSMENT PROCESS

The following section attempts to demonstrate the planning and assessment process by highlighting questions considered by a Grade 7 teacher as he or she plans for mathematics instruction. The questions are expanded upon and supported through concrete examples from *Grades 5 to 8 Mathematics: A Foundation for Implementation* (Manitoba Education and Training) and actual student samples.



I would like to begin working on solving equations in Patterns and Relations.

- In looking at the outcomes, there are three that are closely linked.
  - PR–II.1.7: illustrates the solution process for a one-step, single-variable, first-degree equation, using concrete materials or diagrams
  - PR–II.2.7: solves and verifies one-step linear equations, using a variety of techniques
  - PR–II.3.7: writes mathematical expressions/equations that arise from problem-solving contexts and explains how to solve simple problems, using informal algebraic methods
- These particular outcomes can be taught in an integrated way, rather than in discrete parts, because they are basically dealing with the same concepts.
- At times, it is possible to integrate two or more outcomes, both within the strand itself and between the strands. This integration helps students see the connections between the different areas in mathematics.

Note: The outcomes chosen will depend on the learning sequence or theme being planned. It is important to remain focused on, and to track, the specific learning outcomes for the grade. In this way, teachers can be confident that curriculum expectations are being met.

2. How do the selected outcomes connect to prior and future outcomes.

I notice that in Grade 6, students are balancing equations and using pre-algebra strategies to solve them.

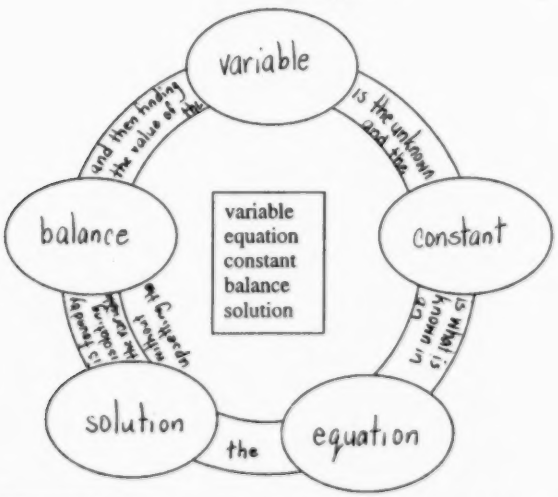
- In Grade 7, they use one-step linear equations.
- In Grade 8, they will be using two-step linear equations.

Grade 6	Grade 7	Grade 8
Uses pre-algebra strategies to solve equations with one unknown and with whole number coefficients and solutions. *(PR-II.2.6)*	Solves and verifies one-step linear equations, using a variety of techniques. *(PR-II.2.7)*	Solves and verifies one- and two-step, first degree equations of the form • $x = a - b$ • $ax = b$ • $x/a = b, a \neq 0$ • $ax = b = c$ • $x/a + b = c, a \neq 0$ where a, b, and c are integers. *(PR-II.2.8)*

- Identify and model new vocabulary that students need to know as they acquire these outcomes.
- Use strategies for introducing, developing, and maintaining vocabulary such as those suggested in *Success For All Learners* (Manitoba Education and Training). Examples include: Word Cycle, Three-Point Approach, Concept Map, and Sort and Predict.

Note: It is important to know how a specific outcome is developed throughout the grade levels. This allows teachers to determine where students may have gaps in their learning and what they already know. It also helps teachers set standards of achievement related to their grade level outcomes by knowing what is expected of students at the next grade levels.

Word Cycle*

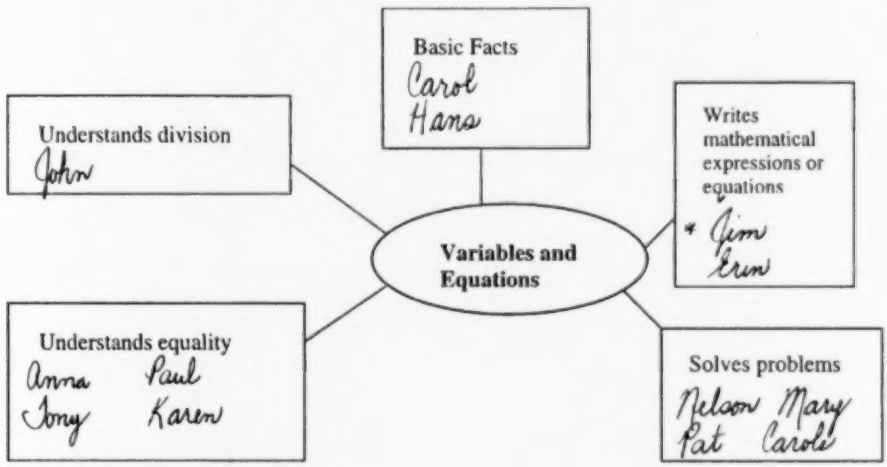


* Adapted from Janice Szabos, G529 *A Novel Approach—Word Cycle*. (Torrance, CA: Good Apple [A Division of Frank Schaffer Publications, 23740 Hawthorne Boulevard, 90505], 1984) 73. Adapted by permission.

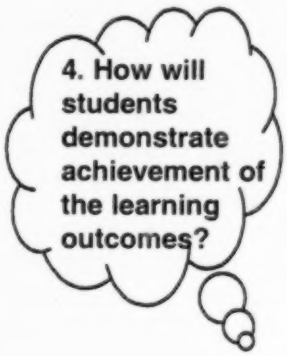
3. What do my students already know?

Rather than giving my students a pretest to determine what they know, I think that I will use the activity on balancing on page B-36 and the two activities listed on page B-38 in *Grades 5 to 8 Mathematics: A Foundation for Implementation* (Manitoba Education and Training). This should provide me with information about what students already know.

- These student learning activities may determine whether or not students understand the preservation of equality when balancing equations. They will also give insight into students' understanding and use of related mathematical language. Most importantly, the activities will allow the teacher to observe students demonstrating their understanding of equality and their ability to "isolate" the unknown. This is also more interesting for students and less threatening than a pretest.
- Use the time during and following the investigation to focus on those students who are missing prior knowledge. Appropriate instructional strategies can be found in *Grades 5 to 8 Mathematics: A Foundation for Implementation* (Manitoba Education and Training).
- Teachers may want to use a map such as the following to note those students who have specific gaps in skills and/or knowledge.



Note: Outcomes in each grade build on prior knowledge; therefore, teachers need to be aware of what students already know, so they can plan appropriate instructional and assessment strategies. This data gathering is most effective if it can be done through a meaningful and engaging activity that requires little or no teacher support. This frees teachers to make observations.



I feel it is important that students know when they have been successful. The class and I will establish performance criteria so that everyone is aware of the expectations. This will allow students to assess their own learning. Perhaps together we could develop a rubric or chart listing expectations. I will ask the students how they will know when they have achieved the outcomes and then post their responses so that they can be used as a basis for self-assessment.

- One goal of assessment is to have students reflect on their learning and to self-assess. In order to help students begin self-assessment they need to be involved in the development of criteria related to the outcome.

  Example: Ask students: "How do you know when you have achieved the outcomes?" Record their responses and then select the key elements for a class chart.

  1. Problem is solved using
     - ❑ materials
     - ❑ diagrams
     - ❑ symbols
  2. Equation is balanced.
  3. Mathematical expressions or equations are written.
  4. Process is explained
     - ❑ orally
     - ❑ in writing
- As students work through the learning task, additional expectations or criteria may be uncovered.
- The Grade 6 standards shown in *Grades 5 to 8 Mathematics: Manitoba Curriculum Framework of Outcomes and Grade 6 Standards* (Manitoba Education and Training) can be used as a model for developing standards at other grade levels.
- It is important that students be able to demonstrate a skill (e.g., solving an equation), but they must also be able to explain how they know they have been successful.

Example: n + 3 = 8

First I got the a cube to represent the variable n and I got centicubes to represent the constants (3 and 8).

I put the cube and 3 centicubes on one side and 8 centicubes on the other. I then removed 3 centicubes from both sides to keep the equation balanced. Now I know that n = 5 because the n is alone on the one side and there are 5 centicubes on the other and the equation is balanced.

n + 3 = 8

n (+ 3 - 3) = 8 - 3

n = 5

Setting Criteria

"Criteria are the standards by which something can be judged or valued" (Gregory, Cameron, and Davies 7).

Setting criteria with students helps them to understand expectations. When expectations are clearly defined, students can use criteria to self-assess. While they are working, they are able to make adjustments based on the criteria and can accomplish the task successfully.

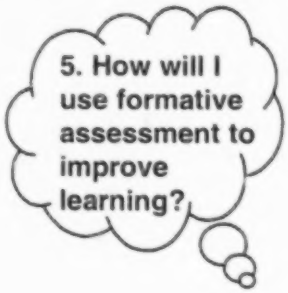
Suggested Steps:

1. **Brainstorm:**
   Pose a question such as "In problem solving, what makes a good answer?" and have students brainstorm possible criteria. Record their responses. Consider taking part in the brainstorming. This helps ensure that the related outcomes are reflected in the criteria.
2. **Sort and Categorize**
   It is important to limit the criteria to three to five points. Have students group the suggestions that fit together. Decide on a heading that best describes the group.
3. **Post**
   Write the criteria clearly on a chart and post it in the classroom where it can be easily seen by all students. If space is not available provide individual or group copies of the criteria.
4. **Add, Revise, Refine**
   Criteria evolve as students learn new concepts and skills.

   Note: When appropriate, have a discussion about the performance criteria that would be "beyond expectations."

The Self-Assessment Process

In order for students to be successful "self-assessors," they need to understand the process. Students must know how they are doing in relation to the outcomes. We want students to be involved in the assessment process. "Self-assessment in education means taking the time to understand who and where one is as a learner. When students reflect on their learning experience, noting what happened, what they learned, how it was different from other learning experiences, what was confusing, in what parts of the experience they had the greatest success, they are engaged in self-assessment" (Moon and Schulman 105). Self-assessment empowers children with reflective strategies that enable them to understand themselves as learners. Teachers need to model the process for their students and have them practise with a partner before getting them to assess themselves.



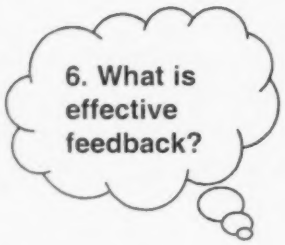
There are many suggestions for assessment gathering and recording in the Foundation document. I think that I will use a checklist to record the data I collect. After I model the process myself, I will also have the students self-check their performance using the class chart.

- Immediate feedback, such as students correcting each other's work based on given criteria or expectations, will improve learning if students are given the opportunity to correct the error(s).
- Students can work in cooperative learning groups. Students within the group can use criteria to measure each other's performance.
- The teacher needs to note
  - — who is struggling and with what expectation(s)
  - — who has met the expectations and who can be challenged
- It is important to allow students that have met the expectations to extend their thinking and learning. In this manner, all students are engaged in meaningful learning activities.
- It is important to document student performance in order to create a picture of student learning over time. It allows the teacher to identify students' strengths and weaknesses. This information helps teachers support learning.

Note: Knowing in advance how this information is going to be gathered helps ensure that the data gathering actually takes place. For example, in order to track observations about student progress related to a particular outcome or group of outcomes, a teacher might use a class list and highlight the students that he or she will observe that day. On the next day, the next group of students is highlighted. In this way, the teacher is able to observe all students over a period of time.

As teachers invest in the documentation process they often

- see patterns emerging within a child's learning style that would otherwise have gone unnoticed
- diagnose student difficulties more effectively, and are then able to provide more appropriate learning experiences
- communicate better with parents about their child's learning by providing specific anecdotal examples
- find it easier to talk with students about their learning because they can relate it to actual classroom events
- find it easier to see the connection between their instruction and student evaluation (assessment) (Moon and Schulman 53–54).



If students are working toward meeting outcomes and have worked together in establishing criteria, should my comments be linked to these in some way? I think that while I am observing the students balancing equations and solving problems I should make comments directing their attention to the criteria.

- Feedback is information that provides students with direct, useable insights into their learning. This information is based on observable differences between what learning they exhibit now, and the learning expected as outlined in an exemplar, goal, or standard.
- The feedback should be matched to the criteria which may be posted on the class chart. Feedback may include sample comments such as the following:
  - — Your diagram clearly shows how you kept the equation balanced.
  - — Your symbols do not match the diagram showing how you solved the equation.
  - — You correctly solved the equation using concrete materials, but how can you make your diagram match what you did?

Effective Stems	Ineffective Stems
I saw …	Good work.
I heard …	Fix that.
How can…	Try harder.

Correcting student work can be an overwhelming task. Teachers need to consider the following:

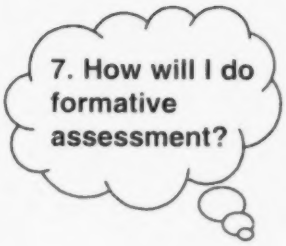
- Determine the purpose of correcting, and follow through.
- Correcting for accountability is fine, but the first accountability is to the child who has done the work, and to his or her improved learning. To achieve the latter, feedback to the child and "feed forward" to the next learning task are what matters.

- The ultimate aim is to offer pupils clear, descriptive feedback that will help them to improve their learning.

Examples:

1. During the last 15 minutes of a class, have students do one question and hand it in to be corrected.
2. On a homework assignment, decide ahead of time what two or three questions will be corrected and focus only on those questions.

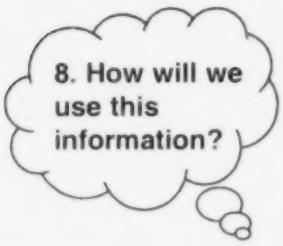
Feedback and correct answers should be given as soon as possible.



I think it is important to use a variety of strategies for my formative assessment. I am going to gather evidence from a combination of self-assessment, peer assessment, and teacher assessment.

- Students can check their work against the criteria and explain how they know they have accomplished outcomes by stating, for example: "I solved the problem using diagrams and symbols. The equation is balanced and I have included an equation. I also explained what I did in writing."
- Students could write a journal/learning log entry explaining how they solve equations. The teacher then could read the description and look for evidence of the criteria.
- Peer assessment is a valuable assessment strategy. Partners should be able to observe each other while balancing and solving equations and make comments based on the criteria. A sample question might be: "You balanced the equation with the materials. Now, can you show what you did using a diagram?"
- To assess skills related to solving problems and one-step linear equations, teachers may choose to observe a student as he or she completes a task. Or, the teacher may conduct an interview with a student by posing the following questions:
  1. I would like you to solve this problem. . .
  2. First, state the mathematical expression or equation you will need
  3. Now, solve it using any materials or method you wish. Teachers may note if the student
     - ❑ uses diagrams
     - ❑ uses symbols
     - ❑ recognizes and corrects errors

Self-assessment behaviours must be modeled. This modeling helps students construct a foundation on which they can build their self-assessment skills. Using a series of questions to guide the self-assessment process provides students with a scaffolding on which to base their own self-assessment. Examples: What did you learn? What was hard for you? Why do think that was hard? What was easy for you? When did we do something like this before? How is this different from what we did before? Hearing these questions will help students see them as a natural extension of any learning experience. Students are then able to ask similar questions of themselves as their self-assessment skills strengthen (adapted from Moon and Schulman 109–110).



My students and I need to look at the information gathered and decide what we need to do next in terms of moving toward meeting the outcomes.

- Together student(s) and the teacher can determine what task(s) are needed to improve learning. Example: The student has immediate opportunities to do more problems related to balancing of equations using materials or solving equations.
- Students can be involved in the recording of their progress related to the outcomes. Teachers do not have to record all of this information.
- Students can set goals for their own learning.

SUMMATIVE ASSESSMENT

1. How will I do summative assessment?

The latest formative assessment(s) seems to indicate that the majority of the students have acquired the skills necessary to meet the three outcomes related to variables and equations. I am now ready for a summative assessment task to determine how well the outcomes have been achieved. Drawing from *Grades 5 to 8 Mathematics: A Foundation for Implementation* (Manitoba Education and Training), I can use the performance task on page B-63 to assess PR.II.1.7 and some questions from B-69 and B-71 to assess the remaining two outcomes. I might even use a structured interview using algebra tiles.

- Summative assessment is usually conducted at the end of a block of instruction and is designed to determine to what extent students have attained learning outcomes. It is primarily used for assigning marks (Manitoba Education and Training, *Reporting on Student Progress and Achievement* 10).
- Summative assessment may take many forms, e.g., paper and pencil tasks, interviews, performance tasks, journal entries, et cetera.
- Performance tasks are tasks designed to provide students with an opportunity to apply mathematical concepts, skills, and processes. While students are engaged in the task, the teacher observes students individually, gathering information about the ways in which they assess personal performance, apply mathematical concepts to the task, solve problems, and make connections with other knowledge and skills.
- Structured interviews used along with a performance task(s) provide teachers with a great deal of information about what a student is able to do. A structured interview usually begins and ends with easy questions. At the beginning, an easy question will put the student at ease, and at the end it allows the student to finish with a sense of accomplishment.

To begin the interview, the teacher might ask the student to set out the following model using algebra tiles:

$x + 3 = -4$

The interview may be conducted by posing questions to the student in the following way:

1) Here is an equation modeled with algebra tiles. Write the equation symbolically. Now, solve the equation and explain your process orally as you work.

— The teacher notes if the student

- ❑ uses zero principle to balance both sides
- ❑ uses correct symbolic equation
- ❑ obtains correct solution

2) Model the equation $3x = 11$ using the algebra tiles. Solve the equation explaining your process orally as you work.

— The teacher should note how the student

- ❑ uses correct model
- ❑ uses groupings
- ❑ obtains correct solution

- Interviews require specific classroom management. While working with one student, the rest of the class should be working independently. Consider having students work on learning activities or assignments in small groups so that group members can field some of the questions normally dealt with by the teacher. Activities chosen should be ones in which the students have had previous experience—not ones that require new teaching.
- To assist with the monitoring of the students involved in activities, help may be provided by
  - — administrators
  - — student teachers
  - — educational assistants
  - — other non-classroom teachers
  - — parent volunteers

Note: The availability of extra help will vary depending on each school's particular situation. In some situations it may be possible to interview two or three students at a time. Two students could sit back to back and the teacher could observe both students as they performed the tasks. The questions could be directed to each student in turn. Three students could be observed if the interview mainly required a hands-on demonstration of skills or concepts.

Creating A Rubric

As students demonstrate the performance, it is appropriate to assess the performance using a rubric. A rubric is a set of scoring guidelines that describe student performance. It is generally used to assess performance tasks and open questions. It can be developed from criteria like that used in formative assessment.

A typical rubric

- contains a scale of points to be assigned: for example, 1 to 4
- describes the characteristics of a response for each possible score

Samples of student work can be used to determine the descriptors for each level of performance. It is important to keep the identity of the students confidential; therefore, it is better if the teacher uses the students' examples by rewriting them on large chart paper.

Using the samples, students can then determine what would be needed to attain each performance level of the rubric for each of the criteria.

Note: When developing a rubric before having student samples or exemplars, it may be necessary to adjust the performance descriptors after looking at student work.

Open Questions

Open questions are designed to prompt students to apply concepts, solve problems, and make mathematical connections. There are no prescribed methods for responding to open-development, open-process, and open-ended questions. Because of this feature, teachers are able to observe the strategies, skills, logic, concepts, and connections students make as they work their way through the questions.

Examples of an

- **open development question:**
  - — Write a challenging story problem for the equation $n - 12 = 65$.
- **open process question:**
  - — The length of a field is 3 times its width. Its perimeter is 4800m. What are the dimensions of the field? Show your work.
- **open-ended question:**
  - — The average mass of 15 pickerel caught in Lake Winnipeg is 2.5 kg. The mode is 3 kg. What are the possible masses of the 15 pickerel? Explain your thinking.

ASSESSING MATHEMATICAL PROCESS

Mental Math

Mental math consists of a collection of strategies that enable a person to estimate, visualize, and manipulate numbers in their heads. Mental math strategies allow students to apply their knowledge of basic facts to compute problems that involve larger numbers.

There are specific methods and procedures that can be taught and practiced the same way that pencil-and-paper algorithms are taught and practiced. Mental math should begin when students enter school and continue through to the end of high school.

In assessing mental math strategies, teachers should look for both oral and written evidence. For example, when asked how they solved the question 55 – 29, a student could answer "I added 1 to the 29 to make 30 and then I added an extra 1 to the 55 to balance. Then I subtracted 56 – 30 and got an answer of 26." The student also could have rounded the 29 to 30 and then adjusted the final answer by adding 1 to 25. Students should be able to describe more than one way to solve a question.

Example:

What mental math strategies could you use to solve this question?

$$\begin{array}{r} 998 \\ +534 \\ \hline \end{array}$$

- "I could solve the question by breaking it up"
  (900 + 500) + (90 + 30) + (8 + 4)
- "I could add 2 to the 998 to get 1000, take 2 away from the 534, and then add 1000 + 532"

Note: Once strategies have been taught, they must be practised. This can be done through number card and number cube games. Teachers can observe the mental math strategies being used as students play the games.

Paper and pencil tests can be used to assess mental math. The tests need to be time-restricted to ensure that students are applying mental math strategies.

Estimation

Estimation is the skill of making a reasonably accurate inference based on prior knowledge or experience. Estimation experiences provide a broad practical context for continued development of children's concept of number, size, and quantity.

In estimating answers to numerical questions, students apply their understanding of place value, mental math strategies, and algorithms. They can use strategies such as

- rounding
- compatibles
- clustering
- front-end
- adjusting

Note: A detailed description of these strategies can be found in the Appendix of *Grades 5 to 8 Mathematics: A Foundation for Implementation* (Manitoba Education and Training).

When estimating size or quantity, students apply their understanding of length, area, capacity/volume, mass, time, money, temperature, and angles. Students need additional strategies for this type of estimation. These strategies include:

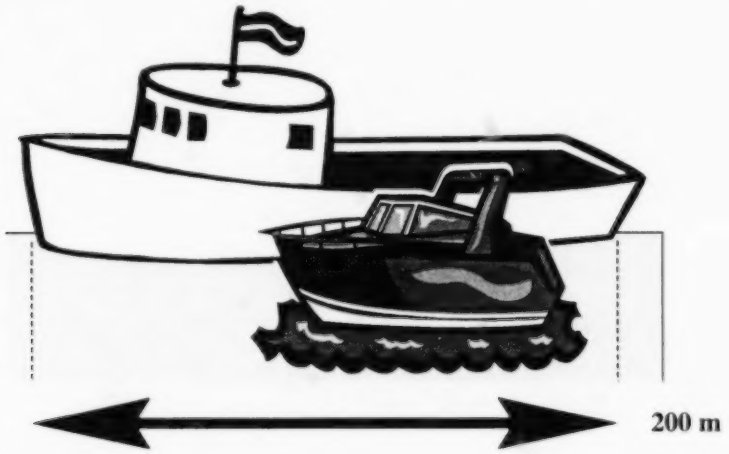
1) **Referents or anchors**: This involves using a known measurement to help students more accurately estimate an unknown measurement. Example: Using the known width of their hand to estimate the length of a table. Note: The referent can also be provided by the teacher.
2) **Chunking**: This involves breaking a measurement problem into smaller problems. Example: Finding the area of the classroom by estimating the cloakroom separately from the main room and adding the two measurements together.
3) **Unitizing**: This involves mentally breaking the entire measurement into smaller parts equal to a known measurement. Example: Pouring 250 mL into a large container, then estimating how many 250 mL units in the whole container to determine the capacity of the container.

Assessing Estimation Skills

Oral Explanation

Example: 12 x 1.9 = ___ "I estimate that the product will be a little less than 24. First I rounded 1.9 to 2. I know that 12 x 2 = 24, but I know that it will be slightly less because 1.9 is less than 2."

Example: If the large boat is 200 m in length, how long is the small boat?



Student Self-Assessment

How many jelly beans fit into a 1 litre container?

My estimate _____

Actual number _____

Difference _____

Next time I will _____

_____

Students can also explain their strategies using pictures, diagrams, and words.

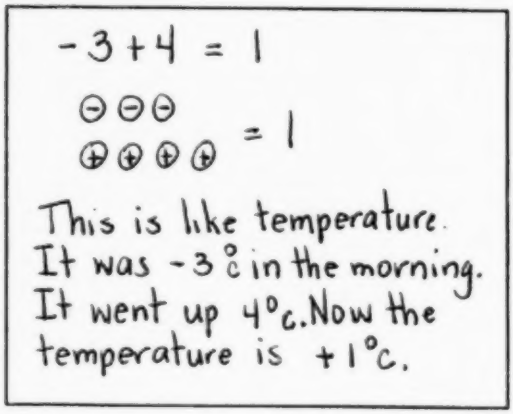
Connections

Students should move beyond understanding a mathematical concept to identifying how mathematical concepts are related to one another, to other subject areas, and to everyday life. A teacher- or student-led discussion may explore concepts relative to mathematics such as

- measurement in industrial arts
- ratio in social studies
- integers in banking
- transformations in art
- collecting and interpreting data and estimating and recognizing patterns in science

Students also need to make connections between the concrete, pictorial, symbolic, oral, and written representations of a concept.

Example:



As an extension, have students complete a Three-Point Approach chart (Manitoba Education and Training, *Success for All Learners* 6.36).

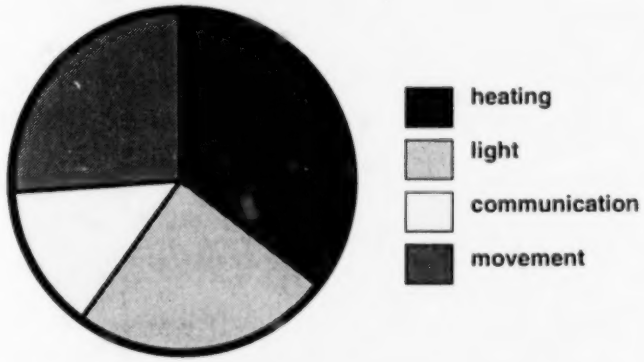
Example:

Definition:	Word or Concept:	Diagram:
_____ _____ _____ _____ _____	Synonym/Example	

Students should be able to transfer and apply their mathematical skills in different situations and contexts. By representing a concept in many different ways, students demonstrate they understand that concept. Teachers can assess students' progress related to connections by using work in other subject areas to collect assessment data for mathematics. Work in other subject areas helps provide students with meaningful contexts in which to apply their mathematical skills.

Example:

Identified Uses of Electricity



Reasoning

Students need to have a real understanding of mathematics. They need to move beyond memorizing sets of rules and procedures, and into investigations that answer the "why" questions. In order to do this, students must be provided with many opportunities to explain, justify, and refine their thinking. Listening to the explanations of their peers and being able to share their own thinking in a safe environment that fosters risk-taking will help them solidify their understandings. Sharing in this way will also help them identify and eliminate some of their misconceptions.

Progress related to reasoning can be assessed by having students construct, illustrate, write, and present their ideas, conceptualizations, and conclusions.

Note: A more detailed description of this will take place in the Communication section.

Problem Solving

Non-routine problem solving is the process in which students apply their understanding of mathematical concepts and skills. This process involves both mathematical investigations and open problems.

Note: Routine problems are generally problems in which the way to a solution is immediately evident. These problems are often "operation questions" presented in word form.

Teachers need to look at four main areas in problem solving and to assess students' progress within each area.

1) **Understanding the problem**: Before students can begin to solve a problem they must be able to interpret its meaning. Teachers can assess students' understanding of the problem by having them do one of the following:
   - rephrase the question in their own words
   - identify the question and draw a diagram
   - highlight the relevant information in one colour and the question in another
   - sort sentence strips from the problem into three groups: needed information, extra information, and the question

2) **Using appropriate strategies**: Students need opportunities to evaluate the effectiveness of using different strategies to address the same problem. In this manner, they are able to move beyond one or two favoured strategies. Teachers can assess the use of strategies by having students explain how they solved the problem. A checklist can be used to assess the explanation and the answer. It may include the following criteria:

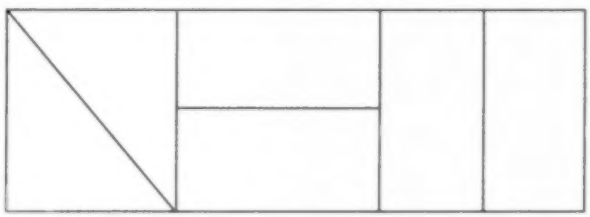
   - ❑ the chosen strategy is appropriate for the question
   - ❑ the strategy is applied correctly
   - ❑ the answer is correct
   - ❑ the explanation is clear
   - ❑ the student states strategy used

3) **Verifying solutions**: Students should self-assess by checking their work or by using another strategy.

4) **Formulating their own problems**: After students have had practice interpreting problems and working with various strategies, they should be able to develop problems of their own. Teachers can assess the problems by using a rubric that is developed together with the students.

An example from Grade 5, page E–57, questions 1 and 3 (Manitoba Education and Training, *Grades 5 to 8 Mathematics: A Foundation for Implementation*):

a) Explain why each piece in this rectangle is one-sixth of its area.



b) A rectangle is three-fifths red. The rest is blue and yellow but not in equal amounts. What could the rectangle look like? Draw **two different** possibilities. (NOTE: This is not an accurate drawing!!)

To assess the previous example, teachers may note if the student

- ❑ explains clearly and correctly how they know that each piece is one-sixth of the rectangle
- ❑ draws two neat and accurate diagrams
- ❑ draws two different diagrams—the fractions for blue and yellow cannot simply be reversed.
- ❑ correctly identifies the fraction of the rectangles that are blue and yellow for each diagram
- ❑ makes sure that the diagrams are reasonably accurate representations of the fractions identified

Scoring Rubric:

Mark	Criteria Met
3	5
2	3-4
1	1-2

Student Sample:

a)

because there are 6 pieces and they all have the same area

b)

R	R	R	B	B
R	R	R	B	Y

3/10 b 1/10 y

Y	R	R	R	B
R	B	R	B	R

3/10 b 1/10 y

R	R	R	B	Y
R	R	R	B	Y
R	R	R	B	Y

4/15 blue 2/15 yellow

This student would score a 3 on the rubric because they have met (and in some cases exceeded) all 5 criteria.

Communication

Students should be able to communicate, both orally and in written form, their mathematical understanding of a problem. The student should be able to use his or her own language to explain and clarify in such a way so that others can understand.

Students should

- use mathematical language and concepts
- explain reasoning
- report evidence
- state a conclusion
- draw and label
- reflect on what they are learning

Example:

There were between 20 and 28 people at a party. The group could not be split into groups of 2, 3, 4 or 5 evenly. How many people were at the party?

Show your work.

~~20~~ ~~21~~ ~~22~~ [23] ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~

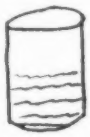
First I crossed out all the even numbers because they are all evenly divisible by 2. Then I crossed out all the numbers that are multiples of 3. The multiples of 4 were already crossed out. Finally I crossed out the multiples of 5 and that left 23. So 23 people attended the party.

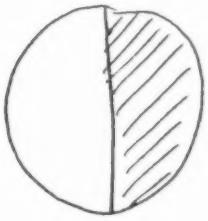
Visualization

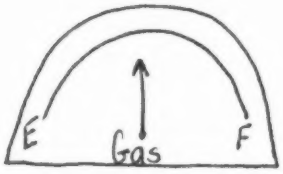
Visualization is the construction of mental models and/or images of mathematical concepts and processes. Visualization of mathematical concepts can be demonstrated by building, drawing, and describing. Students should also be able to identify mathematical concepts in the models and images around them.

Example:

Draw a variety of models showing 50%.







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NOTES